The Elusive Davenport Constant and It's Equally Perplexing Cousin the Cross Number

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Abstract

Let G be a finite abelian group with identity element 0. A finite sequence of elements $M = \{g_1, \ldots, g_k\}$ of not necessarily distinct elements from G is called a *zero-sequence* if $\sum_{i=1}^{k} g_i = 0$. If no proper subsum of M is a zero-sequence, then we call M a *minimal zero-sequence*. Let $\mathcal{U}(G)$ represent the set of minimal zero-sequences of G, and for $M \in \mathcal{U}(G)$, set $\mid M \mid = k$. The *Davenport Constant* of G, denoted by $\mathsf{D}(G)$, is defined by

$$\mathsf{D}(G) = \max\{\mid M \mid : \ M \in \mathcal{U}(G)\}.$$

For $M \in \mathcal{U}(G)$, set

$$\mathsf{k}(M) = \sum_{i=1}^{k} \frac{1}{\mid g_i \mid}.$$

The function k is known as the *cross number* of M, which can be extended to a global constant for G by setting

$$\mathsf{k}(G) = \max\{\mathsf{k}(M) \ : \ M \in \mathcal{U}(G)\}.$$

The computation of both $\mathsf{D}(G)$ and $\mathsf{k}(G)$ has been a frequent topic in the mathematical literature of the past 50 years. We will discuss the history and basic properties of both, as well as their applications in various branches of mathematics. This is a talk intended for a general audience and requires little more than introductory courses in Abstract Algebra and Number Theory.